Algorithmically Verifiable Logic

vis à vis

Algorithmically Computable Logic

Why two complementary Logics could be needed to resolve the EPR paradox

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01: Thesis

If we do not define 'truth' and 'logic' *objectively* then, as Russell quipped:

- ***We*** shall never know what we are talking about, nor whether it is true!

- ***We*** shall simplify the perspective of our thesis by making an arbitrary distinction between the three disciplines:
  - ***Applied science***, whose concern is our sensory observations of a 'common' external world;
  - ***Philosophy***, whose concern is abstracting a coherent perspective of the external world from our sensory observations; and
  - ***Mathematics***, whose concern is adequately expressing such abstractions in a formal language of unambiguous communication.
- ***In*** what follows, our concern is only that of mathematics, where we tentatively suggest that:

Definition

A finite set λ of rules is a Logic of a formal mathematical language $\mathcal{L}$ if, and only if, λ constructively assigns unique truth-values:

> *(**a**) Of provability/unprovability to the formulas of $\mathcal{L}$ which are implied formally by the axioms of $\mathcal{L}$; and*
>
> ***(b)** Of truth/falsity to the sentences of the Theory $T(\mathcal{U})$ which is defined semantically by the λ-interpretation of $\mathcal{L}$ over a structure $\mathcal{U}$.*

03: Overview

- ***The*** motivation for such a definitional approach to 'truth' and 'logic' is that it now allows us to consider the possibility of:
  - ***Adequately*** representing some of the philosophically troubling abstractions of the physical sciences mathematically; and
  - ***Interpreting*** such representations unambiguously.
- ***In*** particular, the perspective we shall suggest is that the paradoxical element which surfaced as a result of the *EPR* argument . . .

- ***Due*** to a putative conflict implied by Bell's inequality between:
    - ***The*** seemingly essential non-locality required by current interpretations of Quantum Mechanics, and
    - ***The*** essential locality required by current interpretations of Classical Mechanics . . .

. . . ***may*** dissolve, if an intelligence could cogently argue:

A: ***That*** all properties of physical reality are deterministic, but their mathematical representations are not necessarily mathematically pre-determined . . .

- ***In*** the sense that any physical property can have one, and only one, value at any time $t(n)$;
- ***Where*** the value is completely determined by some natural law;
- ***Which*** need not, however, be representable mathematically by algorithmically computable functions . . .

  . . . ***and***, therefore, need not be mathematically predictable.

07: Overview (contd.)

B: ***That*** the Laws of Classical Mechanics describe the nature and behaviour of those elements of our physical reality . . .

- ***Whose*** properties are representable mathematically by an algorithmically computable logic, which can be viewed as circumscribing the ambit of reasoning by a mechanical intelligence;
- ***Such*** properties are predictable mathematically since—at any time $t(n)$—they are mathematically pre-determined completely in terms of their putative properties at some earlier time $t(0)$;
- ***The*** values of any two algorithmically computable functions with respect to their variables are, by definition, independent of each other and must, therefore, obey Bell's inequality.

09: Overview (contd.)

C: ***That*** the Laws of Neo-classical Quantum Mechanics describe the nature and behaviour of those elements of our physical reality . . .

10: Overview (contd.)

- ***Whose*** properties are representable mathematically only by an algorithmically verifiable logic, but not by an algorithmically computable logic, which can be viewed as circumscribing the ambit of reasoning by a human intelligence;
- ***Such*** properties are unpredictable mathematically since —at any time $t(n)$—they cannot be theoretically determined completely from only their putative properties at some earlier time $t(0)$;
- ***The*** values of any two algorithmically verifiable functions with respect to their variables may thus be dependent on each other and need not, therefore, obey Bell's inequality.

11: An implicit mathematical ambiguity

The specific thesis of this investigation is that the *EPR* paradox merely reflects an implicit mathematical ambiguity in interpreting quantification, whose roots lie in the assumption of conventional Gödelian wisdom that:

- ***The*** 'true' sentences of a theory $T(\mathcal{U})$ cannot be defined algorithmically by any logic of the formal language $\mathcal{L}$ of the theory $T(\mathcal{U})$,
- ***But*** are an essential feature of the structure $\mathcal{U} =< A, \alpha >$,
- ***Which*** is defined by a non-empty domain A, and an algebra α defined over A.

However we hold that such a non-constructive perspective implicitly implies that the concept of 'truth' must then 'exist' Platonically, in the sense of needing to be discovered by some witness-dependent means—eerily akin to a 'revelation'—if the domain A is infinite.

12: Truth-values must be a computational convention

In this investigation we therefore adopt the constructive perspective that:

- ***The*** 'true' sentences of a theory $T(\mathcal{U})$ must be defined as objective assignments,
- ***By*** a computational convention that is witness-independent,
- ***In*** terms of the Tarskian 'satisfaction' and 'truth' of the corresponding formulas, over the structure $\mathcal{U}$,
- ***Of*** the formal language $\mathcal{L}$ of $T(\mathcal{U})$ under a constructive interpretation.

However, we are then faced with the ambiguity where, if $F^*(x)$ is the interpretation in $\mathcal{U}$ of the $\mathcal{L}$-formula $[F(x)]$ ...

13: Distinguishing between *For any* and *For all*

- ***Is*** the formula $[(\forall x)F(x)]$ of a formal language $\mathcal{L}$ to be interpreted constructively as:
    - ***'For any*** *a*, $F^*(a)$',
    - ***Which*** holds if, and only if,
    - ***For*** *any specified* element *a* of the domain *A*,
    - ***There*** is *algorithmic evidence* that $F^*(a)$ holds in $\mathcal{U}$?

- ***Or*** is $[(\forall x)F(x)]$ to be interpreted finitarily as:
    - ***'For all*** *a*, $F^*(a)$',
    - ***Which*** holds if, and only if,
    - ***There*** is *algorithmic evidence* that,
    - ***For*** *any specified* element *a* of *A*, $F^*(a)$ holds in $\mathcal{U}$?

Where:

Definition

An element *a* of a domain *A* is defined as specifiable in the language $\mathcal{L}$ of a structure $\mathcal{U}$ if, and only if, it can be explicitly denoted as an $\mathcal{L}$-term by an $\mathcal{L}$-formula that interprets as an algorithmically computable constant in $\mathcal{U}$.

14: Evidence-based reasoning

Keeping this distinction in mind, we note that Tarski's classic definitions now:

- ***Permit*** an intelligence,
  - ***Whether*** human,
  - ***Or*** mechanistic,
- ***To*** admit **TWO**
  - ***Finitary,***
  - ***Evidence-based***,
  - ***Inductive,***
- ***Logics*** for assigning
  - ***The*** values of satisfaction and truth,
  - ***To*** the **atomic** formulas of $\mathcal{L}$,
  - ***Over*** the domain A,
- ***In*** **TWO,** essentially different, ways:

(a) In terms of constructive algorithmic verifiabilty; and

(b) In terms of finitary algorithmic computability.

15: Algorithmic verifiability

What this means is that:

- ***If,*** for instance, a number-theoretic formula $[(\forall x)F(x)]$ is to be interpreted constructively, as *'For any* x, $F^*(x)$*'* over the structure $\mathbb{N}$ of the natural numbers,
- ***Then*** it must be read consistently as:

Definition

A number-theoretical relation $F^*(x)$ is algorithmically verifiable if, and only if, for any given natural number n, there is a deterministic algorithm[a] $AL_{(F,\ n)}$ which can provide objective evidence for deciding the truth/falsity of each proposition in the finite sequence $\{F^*(1), F^*(2), \ldots, F^*(n)\}$.

[a] A deterministic algorithm computes a mathematical function which has a unique value for any input in its domain, and the algorithm is a process that produces this particular value as output.

16: Algorithmic computability

Whilst:

- ***If*** the same formula $[(\forall x)F(x)]$ is to be interpreted finitarily, as *'For all x, $F^*(x)$'*,
- ***Then*** it must be read consistently as:

Definition

A number theoretical relation $F^*(x)$ is algorithmically computable if, and only if, there is a deterministic algorithm AL_F that can provide objective evidence for deciding the truth/falsity of each proposition in the denumerable sequence $\{F^*(1), F^*(2), \ldots\}$.

17: Both concepts are constructive

Although both definitions are 'constructive', we note that:

- ***Algorithmic computability*** implies the existence of an algorithm that can witness the truth/falsity of each proposition in a well-defined denumerable sequence of propositions;

Whereas:

- ***Algorithmic verifiability*** does not imply the existence of an algorithm that can witness the truth/falsity of each proposition in a well-defined denumerable sequence of propositions.

Further:

- ***Although*** every algorithmically computable relation is algorithmically verifiable,
- ***The*** converse is false since[1]:

Theorem

There are mathematical functions that are algorithmically verifiable but not algorithmically computable.

[1] *A detailed exposition of these concepts is given in 'The Truth Assignments That Differentiate Human Reasoning From Mechanistic Reasoning: The Evidence-Based Argument for Lucas' Gödelian Thesis.'*

19: Algorithmically computable constants

We note that:

(i): ***All*** the mathematically defined functions known to, and used by, science are algorithmically computable, including those that define transcendental numbers such as π, *e*, etc.

(ii): ***The*** philosophically debatable 'existence' of constants—whether physical or platonic—that are representable mathematically by functions which are algorithmically verifiable, but not algorithmically computable, is suggested by Georg Cantor's diagonal argument.

(iii): ***A*** constructive definition of an arithmetical Boolean function $[R(x)]$ that can be viewed as algorithmically verifiable, but not algorithmically computable, was given by Kurt Gödel in his 1931 paper on formally undecidable arithmetical propositions.

(iv): ***The*** definition of a number-theoretic Halting function that is algorithmically verifiable, but not algorithmically computable, was given by Alan Turing in his 1936 paper on computable numbers.

(v): ***A*** class of Ω constants defined by number-theoretic functions that are algorithmically verifiable, but not algorithmically computable, was given by Gregory Chaitin.

We now consider the significance:

- ***Of*** distinguishing between algorithmically verifiable functions, and algorithmically computable functions,
- ***For*** distinguishing between the mathematical repr-esentations of the laws determining classical and quantum phenomena.

25: The finitary perspective

- ***Functionally*** the difference between the two concepts could be expressed by saying that:
  - ***The*** decimal representation of a real number corresponds to a physically measurable limit . . .

    . . . ***if***, and only if, such representation is definable by an algorithmically computable function.

- ***Now***, we note that at present it is not obvious whether the following postulated dimensionless physical constants can be defined mathematically:
    - α, the fine structure constant.
    - μ or β, the proton-to-electron mass ratio.
    - α_s, the coupling constant for the strong force.
    - αG, the gravitational coupling constant.
- ***Their*** putative values are currently projected only on the basis of physical measurement.

This suggests that:

- **Thesis**: ***Some*** dimensionless physical constants may only be representable in a mathematical language as real numbers that are defined by functions which are algorithmically verifiable, but not algorithmically computable.

If so, we cannot treat such constants as denoting—even in principle—a ‘measurable’ limiting value.

In other words it is conceivable that:

- ***The*** sequence of digits in the decimal representation of an 'unmeasurable' physical constant cannot be treated in a mathematical language as an algorithmically definable infinite sequence;

Whilst:

- ***The*** sequence in the decimal representation of a 'measurable' physical constant can be treated in a mathematical language as an algorithmically definable infinite sequence.

The distinction suggests that:

- ***Any*** physical theory may be mathematically incomplete with respect to the nature and behaviour of some laws of nature that are only representable mathematically by algorithmically verifiable functions.

However it is conceivable that:

- ***Classical*** mechanics can be described as mathematically completable with respect to the nature and behaviour of those laws of nature that are representable mathematically by algorithmically computable functions.

The analogy here is that Gödel has shown in 1931 that:

- ***Any*** formal arithmetic is mathematically incomplete with respect to the algorithmically verifiable nature and behaviour of the natural numbers.

However we have shown at AISB/IACAP Turing 2012 and at Epsilon 2015 that:

- ***The*** first-order Peano Arithmetic PA is finitarily consistent with respect to the algorithmically computable nature and behaviour of the natural numbers . . .
  - . . . from which it follows that PA is complete with respect to the algorithmically computable nature and behaviour of the natural numbers.

31: EPR incompleteness

Viewed from such a perspective, the *EPR* paper could be interpreted as observing presciently:

> *"We are thus forced to conclude that the quantum -mechanical description of physical reality given by wave functions is not (mathematically?) complete."*

The above perspective also suggests that:

- **Thesis**: ***The*** nature and behaviour of two conjugate properties F_1 and F_2 of a particle P may be determined by neo-classical laws that are described mathematically at any time $t(n)$ by two algorithmically verifiable, but not algorithmically computable, functions f_1 and f_2.

33: Conjugate properties

In other words, it may be the very essence of the neo-classical laws determining the nature and behaviour of the conjugate properties F_1 and F_2 of a particle P that, at any time $t(n)$:

- ***We*** can only determine either $f_1(n)$ or $f_2(n)$, but not both;
- ***Hence*** measuring either one makes the other indeterminate if we assume that we cannot go back in time;
- ***This*** does not contradict the assumption that any property of an object must obey some deterministic natural law for any possible measurement that is made at any time.

Such a perspective similarly suggests that:

- **Thesis**: ***The*** nature and behaviour of an entangled property of two particles P and Q may be determined by neo-classical laws that are describable mathematically at any time $t(n)$ by two algorithmically verifiable, but not algorithmically computable, functions f_1 and g_1.

35: Entangled properties

In other words, it may be the very essence of the neo-classical laws determining the nature and behaviour of the conjugate properties F_1 and F_2 of a particle P that, at any time $t(n)$:

- ***Determining*** one immediately gives the state of the other without measurement since the properties are entangled, and cannot be represented mathematically by independent functions;
- ***This*** does not contradict the assumption that any property of an object must obey some deterministic natural law for any possible measurement that is made at any time;
- ***Nor*** does it violate locality by requiring that information travel instantaneously from one particle to another consequent to a measurement.

Further, if $[F(x)]$ is an algorithmically verifiable but not algorithmically computable Boolean function, we can take the query:

- ***Is*** $F(n) = 0$ for all natural numbers?

as corresponding to the Schrödinger question:

- ***Is*** the cat dead or alive at any given time t?

We can then argue that there is no mathematical paradox involved in the assertion that the cat is both dead and alive, if we take this to mean that we may:

- ***Either*** assume the cat to be alive until a given time t (in the future);
- ***Or*** assume the cat to be dead until the time t;

Without arriving at any logical contradiction in our existing Quantum description of nature.

In other words:

- ***Once*** we accept Quantum Theory as a valid description of nature, then there is no mathematical paradox in stating that the theory essentially cannot predict the state of the cat at any moment of future time;
- ***The*** inability to predict the state of the cat at a future time does not arise out of a lack of sufficient information about the laws of the system that Quantum theory is describing, but stems from the very nature of these laws.

39: Mathematical analogy: Independent arithmetical propositions

The mathematical analogy for the above would be that:

- ***Once*** we accept that Peano Arithmetic is consistent and categorical, then we cannot deduce from the axioms of PA for any given arithmetical formula $[F(x)]$ whether $[F(n) = 0]$ for all natural numbers, or whether $[F(n) = 1]$ for some natural number.

To summarise, we argue in this paper that:

- ***All*** properties of physical reality can be deterministic;
- ***The*** Laws of Classical Mechanics would then describe the nature and behaviour of only those deterministic properties that are representable mathematically by algorithmically computable functions—which can be viewed as circumscribing the ambit of reasoning by a mechanical intelligence;
- ***The*** Laws of Quantum Mechanics would then describe the nature and behaviour of those deterministic but essentially unpredictable properties that are representable mathematically only by algorithmically verifiable, but not algorithmically computable, functions—which can be viewed as circumscribing the ambit of reasoning by a human intelligence.

That concludes this presentation

Thank you