A suggested mathematical perspective for the *EPR* argument

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01: Overview

- ***We*** shall simplify the perspective of our paper by making an arbitrary distinction between the three disciplines:
  - ***Applied science***, whose concern is our sensory observations of a 'common' external world;
  - ***Philosophy***, whose concern is abstracting a coherent perspective of the external world from our sensory observations; and
  - ***Mathematics***, whose concern is adequately expressing such abstractions in a formal language of unambiguous communication.
- ***In*** what follows, our concern is only that of mathematics.

02: Overview

- ***Specifically***, we only consider the possibility of:
  - ***Adequately*** representing some of the philosophically troubling abstractions of the physical sciences mathematically; and
  - ***Interpreting*** such representations unambiguously.
- ***In*** particular, the perspective we shall suggest is that the paradoxical element which surfaced as a result of the *EPR* argument . . .

03: Overview

- ***Due*** to a putative conflict implied by Bell's inequality between:
  - ***The*** seemingly essential non-locality required by current interpretations of Quantum Mechanics, and
  - ***The*** essential locality required by current interpretations of Classical Mechanics

04: Overview

. . . ***may*** dissolve if a physicist could cogently argue:

05: Overview (contd.)

A: ***That*** all properties of physical reality are deterministic, but their mathematical representations are not necessarily mathematically pre-determined . . .

06: Overview (contd.)

- ***In*** the sense that any physical property can have one, and only one, value at any time $t(n)$;
- ***Where*** the value is completely determined by some natural law;
- ***Which*** need not, however, be representable mathematically by algorithmically computable functions . . .

  . . . ***and***, therefore, need not be mathematically predictable.

07: Overview (contd.)

B: ***That*** the Laws of Classical Mechanics describe the nature and behaviour of those elements of our physical reality . . .

08: Overview (contd.)

- ***Whose*** properties are representable mathematically by algorithmically computable functions;
- ***Such*** properties are predictable mathematically since—at any time $t(n)$—they are mathematically pre-determined completely in terms of their putative properties at some earlier time $t(0)$;
- ***The*** values of any two such functions with respect to their variables are, by definition, independent of each other and must, therefore, obey Bell's inequality.

09: Overview (contd.)

C: ***That*** the Laws of Neo-classical Quantum Mechanics describe the nature and behaviour of those elements of our physical reality . . .

10: Overview (contd.)

- ***Whose*** properties are representable mathematically only by algorithmically verifiable, but not algorithmically computable, functions;
- ***Such*** properties are unpredictable mathematically since—at any time $t(n)$—they cannot be theoretically determined completely from only their putative properties at some earlier time $t(0)$;
- ***The*** values of any two such functions with respect to their variables may, by definition, be dependent on each other and need not, therefore, obey Bell's inequality.

11: Overview (contd.)

We now consider in detail some remarkable—but hitherto unremarked—mathematical properties of the two critical concepts:

- ***Algorithmic*** verifiability;
- ***Algorithmic*** computability.

12: Algorithmic verifiability

Definition

A number-theoretical relation $F(x)$ is algorithmically verifiable if, and only if, for any given natural number n, there is an algorithm $AL_{(F,\ n)}$ which can provide objective evidence for deciding the truth/falsity of each proposition in the finite sequence $\{F(1), F(2), \ldots, F(n)\}$.

13: Algorithmic computability

Definition

A number theoretical relation $F(x)$ is algorithmically computable if, and only if, there is an algorithm AL_F that can provide objective evidence for deciding the truth/falsity of each proposition in the denumerable sequence $\{F(1), F(2), \ldots\}$.

14: The difference

We note that:

- ***Algorithmic computability*** implies the existence of an algorithm that can decide the truth/falsity of each proposition in a well-defined denumerable sequence of propositions;

Whereas:

- ***Algorithmic verifiability*** does not imply the existence of an algorithm that can decide the truth/falsity of each proposition in a well-defined denumerable sequence of propositions.

15: A theorem

We note that:

- ***Although*** every algorithmically computable relation is algorithmically verifiable,
- ***The*** converse is false.

16: Every algorithmically computable relation is algorithmically verifiable

Theorem

There are mathematical functions that are algorithmically verifiable but not algorithmically computable.

Proof: (a) Since any real number R is mathematically definable as the limit of a Cauchy sequence of rational numbers:

- Let r_n denote the n^{th} digit of the decimal expression—in binary notation ($0 \leq r_n \leq 1$)—of the real number:

$$R = Lt_{n\rightarrow\infty} \sum_{k=1}^{n} r_k.10^{-k} = 0.r_1 r_2 \ldots r_n \ldots$$

- Since any finite sequence is recursive trivially, for any given natural number n there is an algorithm $AL_{(R,\ n)}$ that can decide the truth/falsity of each proposition in the finite sequence $\{r_1 = 0, r_2 = 0, \ldots, r_n = 0\}$.
- Hence, for any real number R, the relation $r_n = 0$ is algorithmically verifiable trivially.

17: The converse is false

Proof: (b) Since it follows from Alan Turing's Halting argument that there is an algorithmically uncomputable real number H:

- Let h_n denote the n^{th} digit of the decimal expression in binary notation of the algorithmically *uncomputable* real number $H = 0.h_1 h_2 \ldots h_n \ldots$.
- By (a), the relation $[h_n = 0]$ is algorithmically verifiable trivially.
- However, by definition there is no algorithm AL_H that can decide the truth/falsity of each proposition in the denumerable sequence:

  $\{[h_1 = 0], [h_2 = 0], \ldots\}$.
- Hence the relation $[h_n = 0]$ is algorithmically verifiable but not algorithmically computable. □

18: The finitary perspective

- ***Finitarily*** the difference between the two concepts could be expressed by saying that:
  - ***The*** decimal representation of a real number corresponds to a physically measurable limit . . .
    - ***As*** required to resolve Zeno's paradox,

  . . . ***if***, and only if, such representation is definable by an algorithmically computable function.

19: Algorithmically computable constants

We note that:

(i): ***All*** the mathematically defined functions known to, and used by, science are algorithmically computable, including those that define transcendental numbers such as π, *e*, etc.

20: Cantor's diagonal argument

(ii): ***The*** philosophically debatable 'existence' of constants—whether physical or platonic—that are representable mathematically by functions which are algorithmically verifiable, but not algorithmically computable, is suggested by Georg Cantor's diagonal argument.

21: Gödel's undecidable arithmetical proposition

(iii): ***A*** constructive definition of an arithmetical Boolean function $[R(x)]$ that can be viewed as algorithmically verifiable, but not algorithmically computable, was given by Kurt Gödel in his 1931 paper on formally undecidable arithmetical propositions.

22: Turing's Halting function

(iv): ***The*** definition of a number-theoretic Halting function that is algorithmically verifiable, but not algorithmically computable, was given by Alan Turing in his 1936 paper on computable numbers.

23: Chaitin's Ω constants

(v): ***A*** class of Ω constants defined by number-theoretic functions that are algorithmically verifiable, but not algorithmically computable, was given by Gregory Chaitin.

24: 'Uncomputable' physical constants

- ***Now***, we note that at present it is not obvious whether the following postulated dimensionless physical constants can be defined mathematically:
  - α, the fine structure constant.
  - μ or β, the proton-to-electron mass ratio.
  - α_s, the coupling constant for the strong force.
  - αG, the gravitational coupling constant.
- ***Their*** putative values are currently projected only on the basis of physical measurement.

25: 'Uncomputable' physical constants are 'unmeasurable'

This suggests that:

- **Thesis**: ***Some*** dimensionless physical constants may only be representable in a mathematical language as real numbers that are defined by functions which are algorithmically verifiable, but not algorithmically computable.

If so, we cannot treat such constants as denoting—even in principle—a 'measurable' limiting value.

26: Algorithmically definable and algorithmically undefinable infinite sequences

In other words it is conceivable that:

- ***The*** sequence of digits in the decimal representation of an 'unmeasurable' physical constant cannot be treated in a mathematical language as an algorithmically definable infinite sequence;

Whilst:

- ***The*** sequence in the decimal representation of a 'measurable' physical constant can be treated in a mathematical language as a algorithmically definable infinite sequence.

27: Our description of natural laws is mathematically incomplete

The distinction suggests that:

- ***Any*** physical theory may be mathematically incomplete with respect to the nature and behaviour of some laws of nature that are only representable mathematically by algorithmically verifiable functions.

However it is conceivable that:

- ***Classical*** mechanics can be described as mathematically completable with respect to the nature and behaviour of those laws of nature that are representable mathematically by algorithmically computable functions.

28: Our description of natural numbers is algorithmically incomplete

The analogy here is that Gödel showed in 1931 that:

- ***Any*** formal arithmetic is mathematically incomplete with respect to the algorithmically verifiable nature and behaviour of the natural numbers.

However we have shown at AISB/IACAP Turing 2012 that

- ***The*** first-order Peano Arithmetic PA is finitarily consistent with respect to the algorithmically computable nature and behaviour of the natural numbers . . .
  - . . . from which it follows that PA is complete with respect to the algorithmically computable nature and behaviour of the natural numbers.

29: EPR incompleteness

Viewed from such a perspective, the *EPR* paper could be interpreted as observing presciently:

> *"We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not (mathematically?) complete."*

30: Conjugate properties

The above perspective also suggests that:

- **Thesis**: ***The*** nature and behaviour of two conjugate properties F_1 and F_2 of a particle P may be determined by neo-classical laws that are described mathematically at any time $t(n)$ by two algorithmically verifiable, but not algorithmically computable, functions f_1 and f_2.

31: Conjugate properties

In other words, it may be the very essence of the neo-classical laws determining the nature and behaviour of the conjugate properties F_1 and F_2 of a particle P that, at any time $t(n)$:

- ***We*** can only determine either $f_1(n)$ or $f_2(n)$, but not both;
- ***Hence*** measuring either one makes the other indeterminate if we assume that we cannot go back in time;
- ***This*** does not contradict the assumption that any property of an object must obey some deterministic natural law for any possible measurement that is made at any time.

32: Entangled properties

Such a perspective similarly suggests that:

- **Thesis**: ***The*** nature and behaviour of an entangled property of two particles P and Q may be determined by neo-classical laws that are describable mathematically at any time $t(n)$ by two algorithmically verifiable, but not algorithmically computable, functions f_1 and g_1.

33: Entangled properties

In other words, it may be the very essence of the neo-classical laws determining the nature and behaviour of the conjugate properties F_1 and F_2 of a particle P that, at any time $t(n)$:

- ***Determining*** one immediately gives the state of the other without measurement since the properties are entangled, and cannot be represented mathematically by independent functions;
- ***This*** does not contradict the assumption that any property of an object must obey some deterministic natural law for any possible measurement that is made at any time;
- ***Nor*** does it require any information to travel from one particle to another consequent to a measurement.

34: Schrödinger's cat

Further, if $[F(x)]$ is an algorithmically verifiable but not algorithmically computable Boolean function, we can take the query:

- ***Is*** $F(n) = 0$ for all natural numbers?

as corresponding to the Schrödinger question:

- ***Is*** the cat dead or alive at any given time t?

35: Schrödinger's statement independent of algorithmic laws

We can then argue that there is no mathematical paradox involved in the assertion that the cat is both dead and alive, if we take this to mean that we may:

- ***Either*** assume the cat to be alive until a given time t (in the future);
- ***Or*** assume the cat to be dead until the time t;

without arriving at any logical contradiction in our existing Quantum description of nature.

36: Schrödinger's statement independent of algorithmic laws

In other words:

- ***Once*** we accept Quantum Theory as a valid description of nature, then there is no mathematical paradox in stating that the theory essentially cannot predict the state of the cat at any moment of future time;
- ***The*** inability to predict the state of the cat at a future time does not arise out of a lack of sufficient information about the laws of the system that Quantum theory is describing, but stems from the very nature of these laws.

37: Mathematical analogy: Independent arithmetical propositions

The mathematical analogy for the above would be:

- ***Once*** we accept that Peano Arithmetic is consistent and categorical, then we cannot deduce from the axioms of PA for any given arithmetical formula $[F(x)]$ whether $[F(n) = 0]$ for all natural numbers, or whether $[F(n) = 1]$ for some natural number.

38: Summary

To summarise, we argue in this paper that:

- ***All*** properties of physical reality can be deterministic;
- ***The*** Laws of Classical Mechanics would then describe the nature and behaviour of only those deterministic properties that are representable mathematically by algorithmically computable functions;
- ***The*** Laws of Quantum Mechanics would then describe the nature and behaviour of those deterministic but essentially unpredictable properties that are representable mathematically only by algorithmically verifiable, but not algorithmically computable, functions.

End

That concludes this presentation

Thank you